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SEQUENCES AND SERIES

DEFINITIONS AND FORMULAS

A Sequence *is a list of values separated by commas.*
A Series *is the indicated sum of a sequence; the commas are replaced by plus signs.*
A term *is any of the individual values of a sequence or series.*

ARITHMETIC, GEOMETRIC, AND RECURSIVE

Definition	Formulas: n^{th} term	Formulas: n^{th} sum	Formulas: infinite sum
Arithmetic Sequences and Series *An arithmetic sequence has a* ***common difference d*** *between each successive term given by*: $a_{n+1} - a_n$.	$a_n = a_1 + (n-1)d$	$A_n = n\left(\frac{a_1 + a_n}{2}\right)$	D.N.E.
Geometric Sequences and Series *A geometric sequence has a* ***common ratio r*** *between successive terms given by*: g_{n+1} / g_n.	$g_n = g_1 r^{n-1}$	$G_n = \frac{g_1(1-r^n)}{1-r}$	$G = \frac{g_1}{1-r}$
Recursive Sequences and Series *A recursive sequence has a* ***common formula*** *involving preceding term(s), between successive terms, given as* $f(t_{n-1,2,...})$.			

SIGMA NOTATION

Definitions

Sigma operator: add all f(k)'s in domain a to b.

$$\sum_{k=a}^{b} f(k)$$

- b: *Last number in domain*
- $f(k)$: *Object function*
- a: *First number of domain*
- k: *Index*

Properties

constant

$$\sum_{k=a}^{b} cf(k) = c\sum_{k=a}^{b} f(k)$$

additive

$$\sum_{k=a}^{b} f(k) \pm g(k) = \sum_{k=a}^{b} f(k) \pm \sum_{k=a}^{b} g(k)$$

FINDING AN n^{th} TERM EXPRESSION

1) *Do the terms have a* ***common difference d****? Use the formula above for the* n^{th} *term of an arithmetic sequence.*
2) *Do the terms have a* ***common ratio r****? Use the formula above for the* n^{th} *term of a geometric sequence.*
3) *Is it a common sequence (see lower right), or some variation thereof?*
4) *Write out the sequence left to right; number above each term with the value of n for that term; look for a pattern between the n's and the terms of the sequence.*
5) *Use* $(-1)^n$ *or* $(-1)^{n-1}$ *for alternating signs between terms.*

Convergence

A ***sequence*** *is said to converge if its individual terms approach some discreet value: a sequence only converges if the value of its even terms and the value of its odd terms both approach the same value.*

A ***series*** *is said to converge if its sum approaches some discreet value; the more terms that are added, the closer the sum converges to its limit (infinite terms = infinitely close): geometric series with* $|\mathbf{r}| < 1$ *always converge.*

MacLaurin Series

A function may be approximated about any point x_0 *by an appropriate series known as a Taylor series. If* $x_0 = 0$ *then the series is known as a MacLaurin series. The MacLaurin series approximations for some important functions are shown below.*

pi

$$\pi = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

e^x

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

sin x

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

cos x

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

binomial theorem

$$(1+x)^m = \sum_{k=0}^{\infty} \frac{m(m-1)\cdots(m-k+1)}{k!} x^k$$

Common Sequences

1,2,3,4,5,6...	arithm: $d = 1$ recur: $t_{n-1}+1$
1,3,5,7,9,11...	arithm: $d = 2$ recur: $t_{n-1}+2$
2,4,6,8,10,12...	arithm: $d = 2$ recur: $t_{n-1}+2$
2,4,8,16,32,64...	geom: $r = 2$ recur: $2t_{n-1}$
1,10,100,1000...	geom: $r = 10$ recur: $10t_{n-1}$
1,2,3,5,8,13...	recur: $t_{n-2} + t_{n-1}$
1,4,9,16,25,36...	n^2
1,8,27,64,125,216...	n^3
1,2,6,24,120,720...	$n!$
2,3,5,7,11,13,17,19...	prime #'s