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Sketching Polynomials

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For any Polynomial

$$P(x) = \pm a_n x^n \pm a_{n-1} x^{n-1} \pm \cdots \pm a_1 x^1 \pm a_0$$

Phast Phacts: a birdseye view of the process

1. *What is the order?*
2. *Symmetry of parent function*
3. *Reflections* $\pm a_n$
4. *y-intercept: constant*
5. *x- intercepts: roots, zeros*
6. *Transforms and multiplicities*

Housekeeping

simplify -
remove grouping symbols.
remove common factors.
combine like terms.

put in standard form -
descending order from left.
all orders must appear.
set equal to zero.

First term

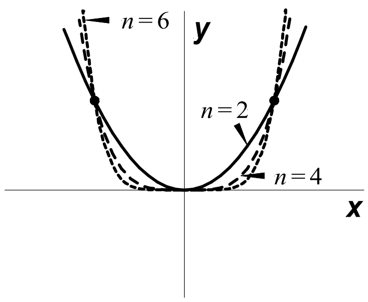
the order is given by n -
n gives the total number of roots.
n – 1 gives the maximum number of turning points.
symmetry(parent functions)-
n even: left/right symmetry.
n odd: origin symmetry.
reflections -
$\pm a_n$ *reflects across x-axis.*

Remember

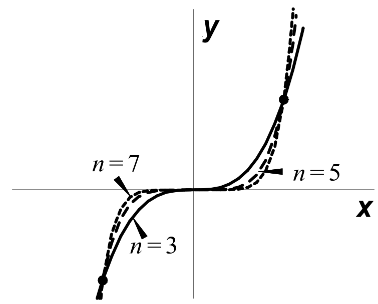
y-intercept -
the constant term a_0 *is the y-intercept.*
location theorem -
a zero must lie between +'ve and –'ve remainders.
conjugate pair theorem -
complex roots come in conjugate pairs.

Parent Functions

The functions are shown in "standard position" with exactly one real root (zero "crossing") each.



even functions



odd functions

Real and Rational roots

Descarte's rule of signs: *Substitute any +'ve or -'ve value of x, the resulting number of sign changes is equal to the number of +'ve or -'ve real zeros respectively, or less by a factor of 2.*

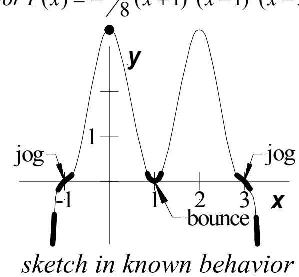
Rational root theorem: *Divide every factor, both +'ve and -'ve, of the constant term* a_0 *by every factor of the* n^{th} *term coefficient* a_n*; the resulting list is every possible rational root, this does not include irrational or complex roots!*

Synthetic division

remainders: *potential roots & remainders form (x, y) pairs.*
upper bound for roots: *all sums have same sign.*
lower bound for roots: *sums have alternating signs.*
****to be sure all roots are found finish with the quadratic eq.*

Sketching

for $P(x) = -\frac{1}{8}(x+1)^3(x-1)^2(x-3)^3$



sketch in known behavior and connect the dots

Transforms & Multiplicities

Arrange factored form as:

$$P(x) = \pm a(x - r_1)^m (x - r_2)^n + k$$

transformations
reflection: $\pm$ *scale: a*
horiz. shift: r_n *vert. shift: k*

multiplicities
m or n even: bounce at r + k
m or n odd: jog at r + k