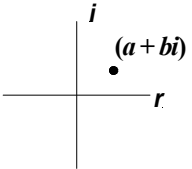


POLAR FUNCTIONS

Complex Trigonometry

<p>Complex numbers $z = a + bi$ $r(\cos \theta + i \sin \theta) = r \text{ cis } \theta$</p>	<p>Complex Coordinates Axes real axis: r imaginary axis: i</p>	
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<p>rectangular to polar $r = \sqrt{x^2 + y^2}, \tan \theta = \frac{y}{x}$</p>	<p>Conversions</p>	<p>polar to rectangular $x = r \cos \theta, \quad y = r \sin \theta$</p>
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Complex polar arithmetic

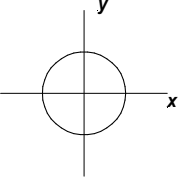
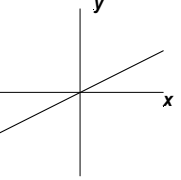
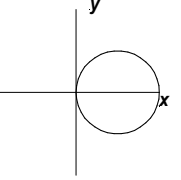
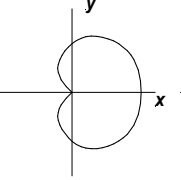
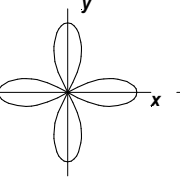
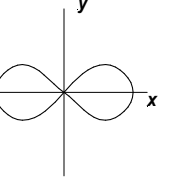
<p>products $z_1 z_2 = r_1 r_2 \text{ cis } (\theta_1 + \theta_2)$</p> <p>quotients $\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{ cis } (\theta_1 - \theta_2)$</p>	<p>powers $(r \text{ cis } \theta)^n = r^n \text{ cis } n\theta$</p> <p>roots $(r \text{ cis } \theta)^{1/n} = r^{1/n} \text{ cis } \left(\frac{\theta + 2\pi k}{n} \right), k = 0, 1, \dots, n-1$</p>
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Graphing

<p><u>Symmetry tests</u></p> <p>horizontal axis $\cos \theta = \cos (-\theta)$</p> <p>vertical axis $\sin \theta = \sin (\pi - \theta)$</p> <p>origin $r \Rightarrow -r$</p>	<p><u>Test points</u></p> <p>r-max, mins, zeros</p> <table style="margin: auto;"> <tr> <th>deg*</th> <th>rad*</th> </tr> <tr> <td>0°</td> <td>0</td> </tr> <tr> <td>90°</td> <td>$\pi/2$</td> </tr> <tr> <td>180°</td> <td>π</td> </tr> <tr> <td>270°</td> <td>$3\pi/2$</td> </tr> </table> <p>*Sometimes in multiples of 1/n</p>	deg*	rad*	0°	0	90°	$\pi/2$	180°	π	270°	$3\pi/2$	<p><u>T-table</u></p> <p>special angles</p> <table style="margin: auto;"> <tr> <td>$\sin 30 = 1/2$</td> </tr> <tr> <td>$\cos 60 = 1/2$</td> </tr> <tr> <td>$45^\circ = \pi/4 \text{ rad}$</td> </tr> </table>	$ \sin 30 = 1/2$	$ \cos 60 = 1/2$	$45^\circ = \pi/4 \text{ rad}$
deg*	rad*														
0°	0														
90°	$\pi/2$														
180°	π														
270°	$3\pi/2$														
$ \sin 30 = 1/2$															
$ \cos 60 = 1/2$															
$45^\circ = \pi/4 \text{ rad}$															

Parent functions

Symmetry rotates counterclockwise QI – QIV, as a function of cos, sin, -cos, -sin, respectively.

					
$r = a$	$\theta = m$	$r = 2a \cos \theta$	$r = a(1 + \cos \theta)$	$r = a \cos n\theta$ $n_{\text{odd}} = n \text{ petals}$ $n_{\text{even}} = 2n \text{ petals}$	$r^2 = a^2 \cos 2\theta$