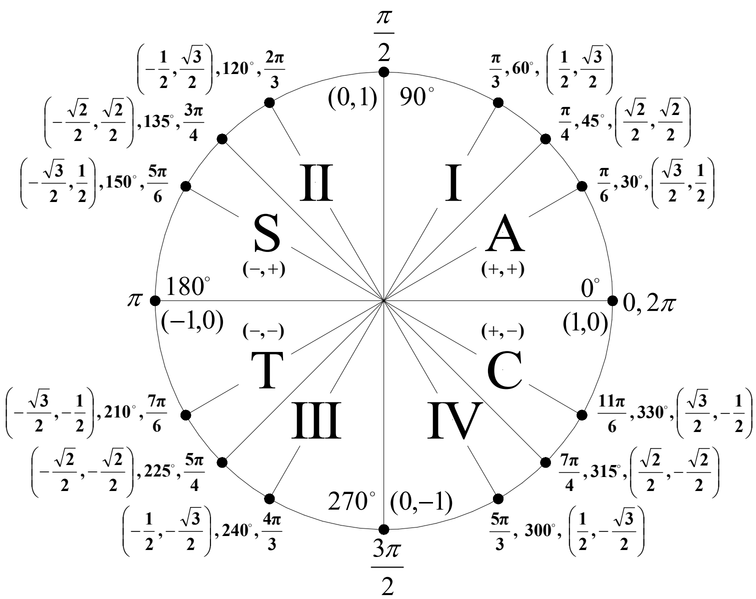
PHYSICS PHOR PHUN
PASO ROBLES, CA

CIRCLE TRIGONOMETRY

MATH AND SCIENCE TUTORING
805-610-1725

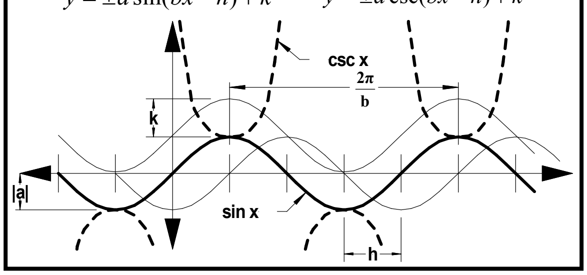
The Unit Circle



Graphs of Trig Functions

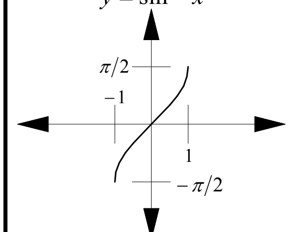
sin and csc

$y=\pm a\sin(bx-h)+k$ $\quad y=\pm a\csc(bx-h)+k$



inverse sin

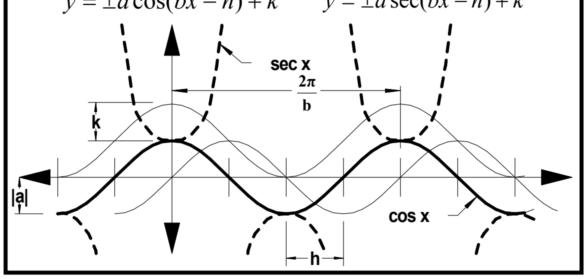
$y=\sin^{-1}x$



Domain: $-1\le x\le 1$

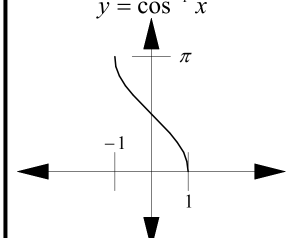
cos and sec

$y=\pm a\cos(bx-h)+k$ $\quad y=\pm a\sec(bx-h)+k$



inverse cos

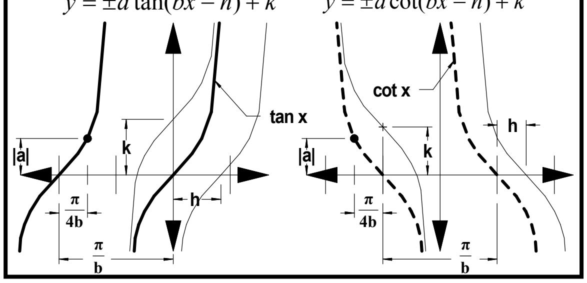
$y=\cos^{-1}x$



Domain: $-1\le x\le 1$

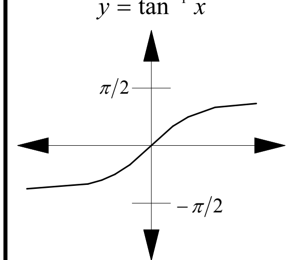
tan and cot

$y=\pm a\tan(bx-h)+k$ $\quad y=\pm a\cot(bx-h)+k$



inverse tan

$y=\tan^{-1}x$



Identities

Proving identities: **1.** *Put everything in terms of sin and cos.* **2.** *Work either side of, but never across, the equal sign.* **3.** *Fractions that equal 1, and conjugation, i.e.,* x^2-y^2, *are often useful tools.*

Pythagorean

$$\sin^2 x+\cos^2 x=1$$
$$\tan^2 x+1=\sec^2 x$$
$$1+\cot^2 x=\csc^2 x$$

ratio

$$\tan x=\frac{\sin x}{\cos x} \qquad \cot x=\frac{\cos x}{\sin x}$$

compliment

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x$$
$$\csc\left(\frac{\pi}{2}-x\right)=\sec x$$
$$\cos\left(\frac{\pi}{2}-x\right)=\sin x$$
$$\sec\left(\frac{\pi}{2}-x\right)=\csc x$$
$$\tan\left(\frac{\pi}{2}-x\right)=\cot x$$
$$\cot\left(\frac{\pi}{2}-x\right)=\tan x$$

negative

$$\sin(-x)=-\sin x$$
$$\csc(-x)=-\csc x$$
$$\cos(-x)=\cos x$$
$$\sec(-x)=\sec x$$
$$\tan(-x)=-\tan x$$
$$\cot(-x)=-\cot x$$

half-angle

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}$$
$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2}$$
$$\tan^2\frac{x}{2}=\frac{1-\cos x}{1+\cos x}$$

supplement

$$\sin(\pi\pm x)=\mp\sin x$$
$$\csc(\pi\pm x)=\mp\csc x$$
$$\cos(\pi\pm x)=-\cos x$$
$$\sec(\pi\pm x)=-\sec x$$
$$\tan(\pi\pm x)=\pm\tan x$$
$$\cot(\pi\pm x)=\pm\cot x$$

double-angle

$$\sin 2x=2\sin x\cos x$$
$$\cos 2x=\cos^2 x-\sin^2 x$$
$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$$

angle sum and difference

$$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$$
$$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$$
$$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$$

addition and subtraction

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x-\cos y=2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

product

$$\sin x\sin y=\frac{\cos(x-y)-\cos(x+y)}{2}$$
$$\cos x\cos y=\frac{\cos(x-y)+\cos(x+y)}{2}$$
$$\sin x\cos y=\frac{\sin(x+y)+\sin(x-y)}{2}$$
$$\cos x\sin y=\frac{\sin(x+y)-\cos(x-y)}{2}$$